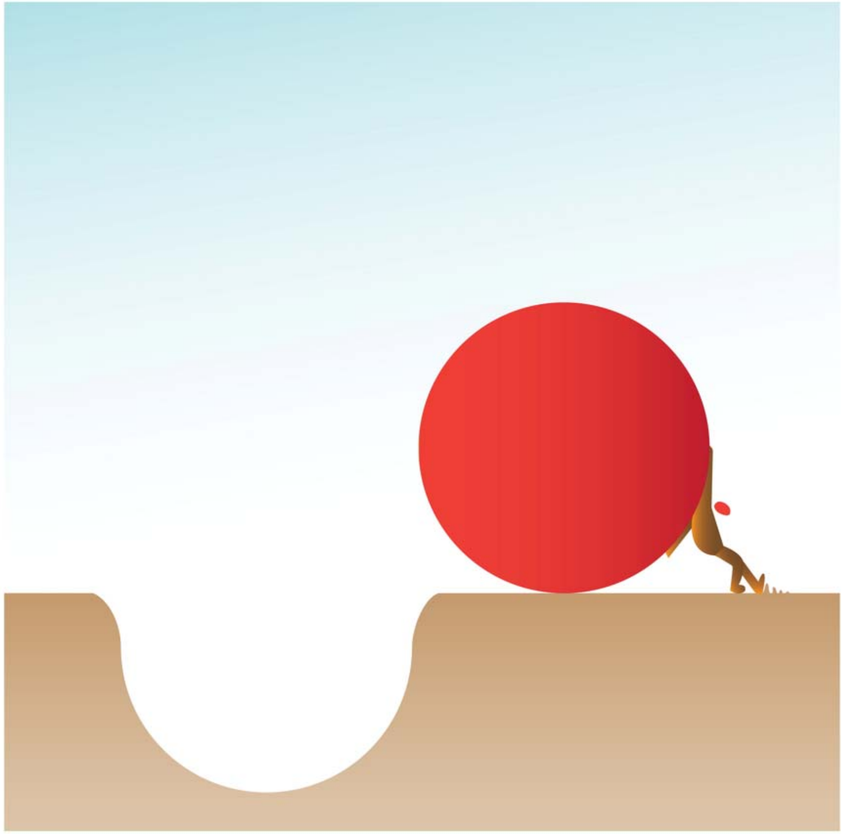
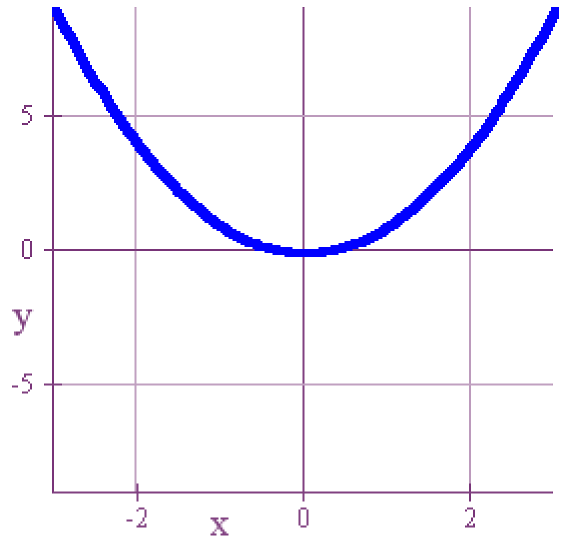
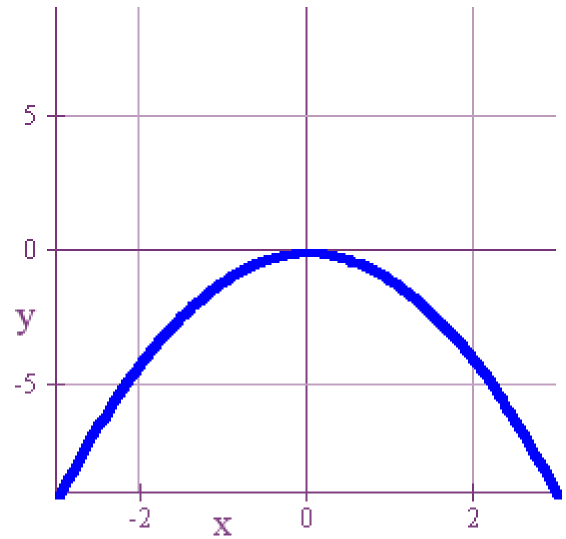
CONCAVITY



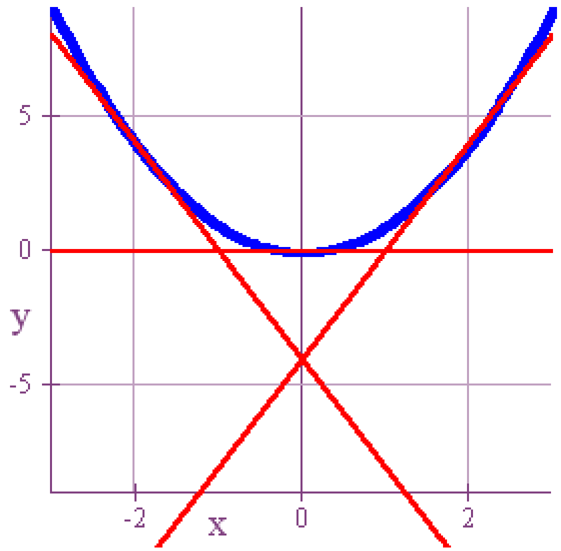
A curve that looks like it will "hold water" is called concave up.



A curve that looks like it will "spill water," is called concave down.



Notice that if we add some tangent lines to our concave up graph, then the slopes of the tangent lines are increasing.



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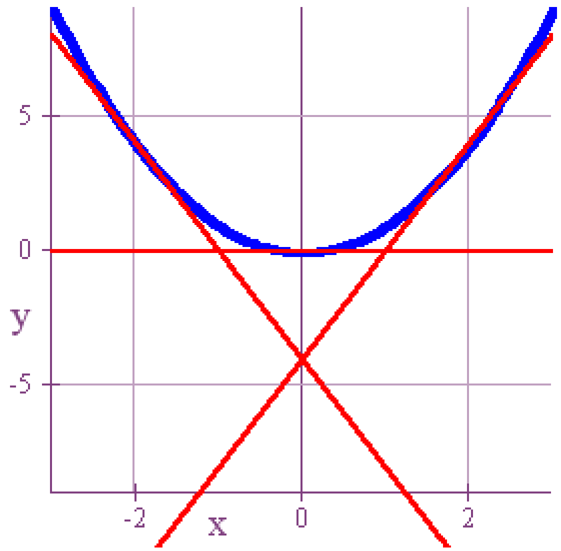
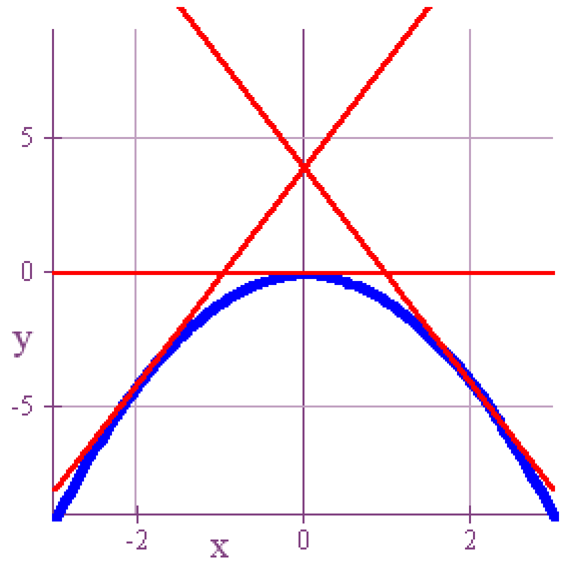


In other words, $f'(x)$ is increasing which means that its derivative is positive.

Similarly, if we add tangent lines to our concave down graph, then the slopes of the tangent lines are decreasing.



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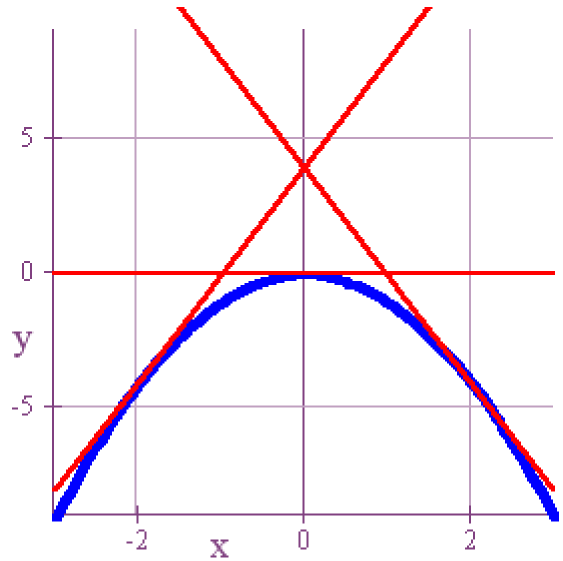

In other words, $f'(x)$ is decreasing which means that its derivative is negative.

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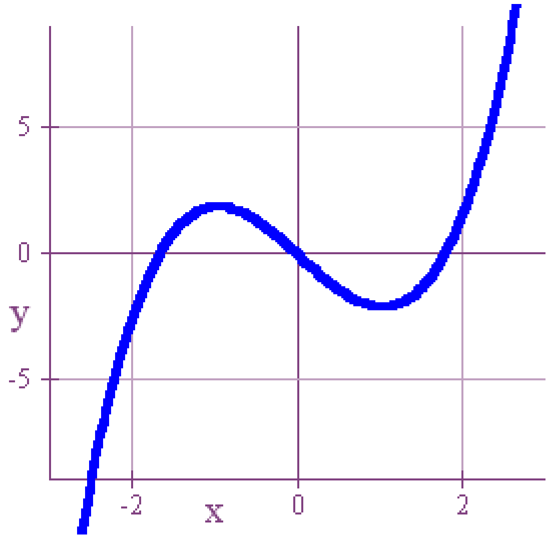
Here's an example.

$$f(x) = x^3 - 3x$$

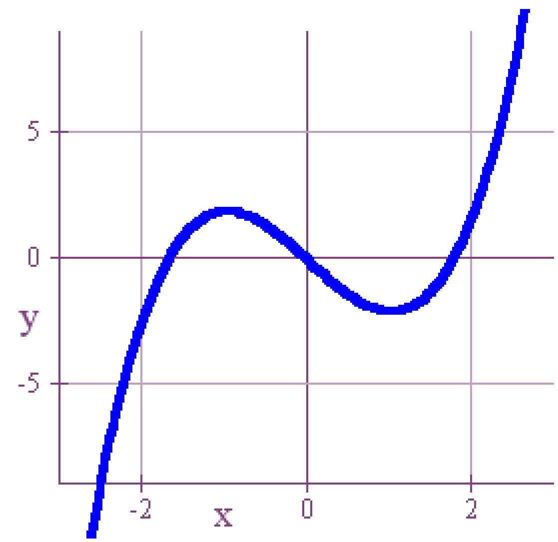


Here's an example.

$$f(x) = x^3 - 3x$$

$$f'(x) = 3x^2 - 3$$

$$f''(x) = 6x$$



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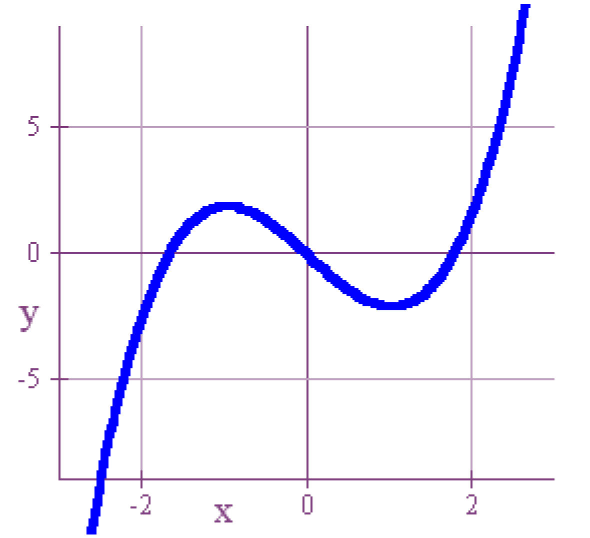

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$$f(x) \text{ is concave up if } x > 0$$

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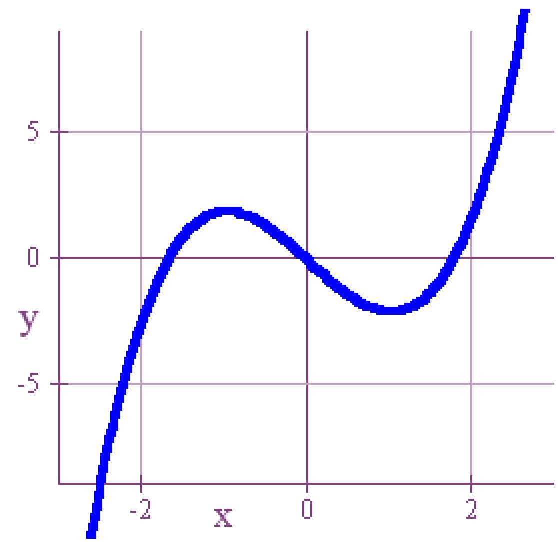

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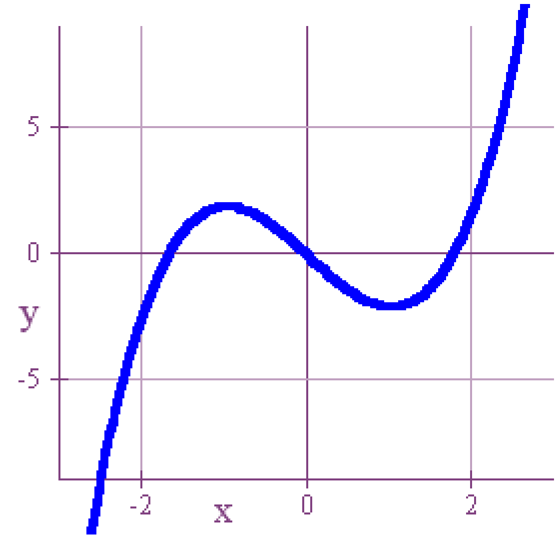

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The point $(0,0)$ is called an inflection point since the concavity changes at this point.