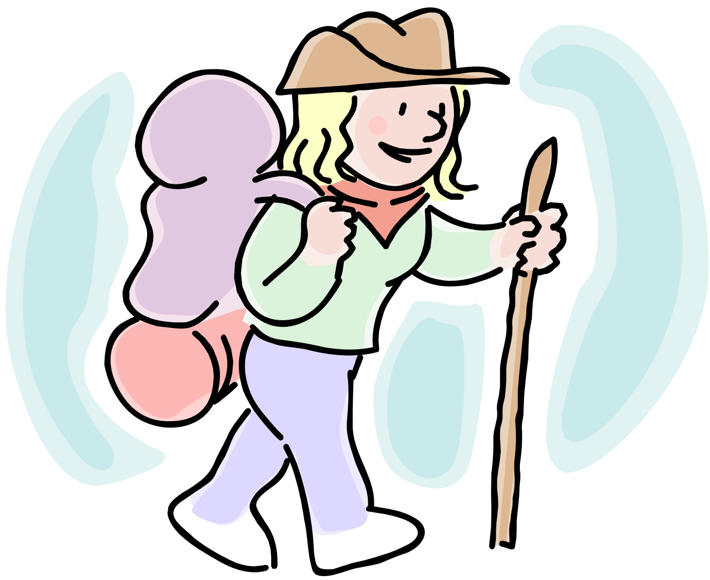
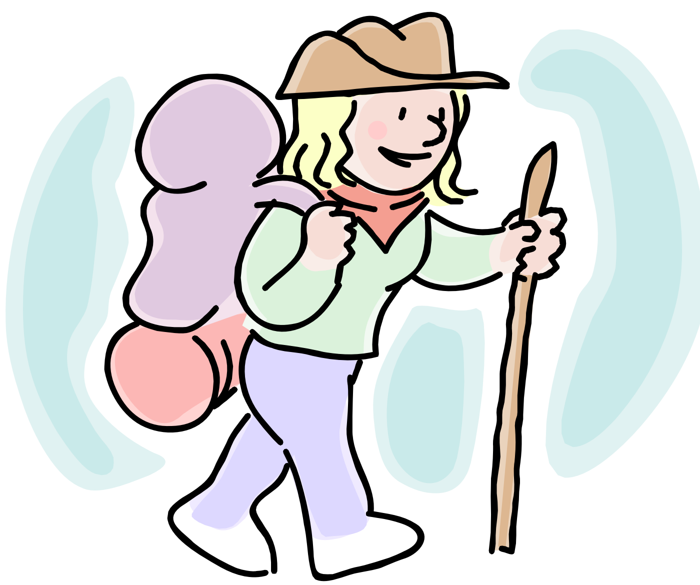
Average Rate of Change

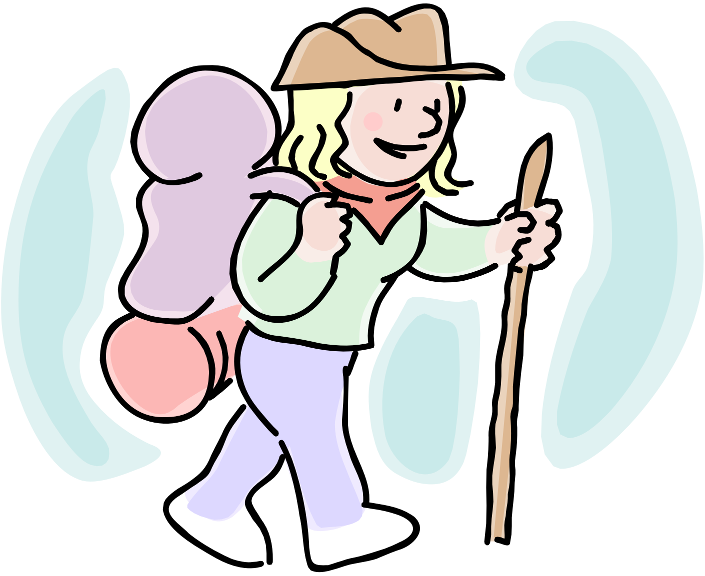


Suppose you go on a walk, and after two hours you have gone four miles. What is your average velocity?



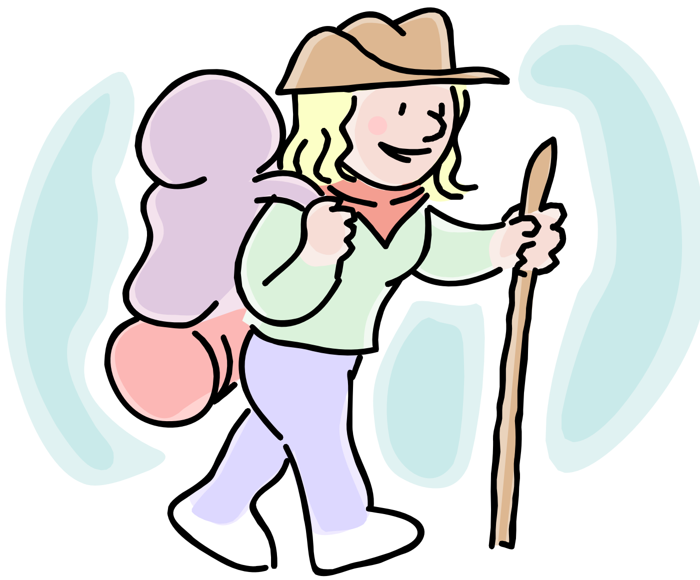
Suppose you go on a walk, and after two hours you have gone four miles. What is your average velocity?

$$\text{average velocity} = \frac{4\text{ miles}}{2\text{ hours}} = 2\,\frac{\text{miles}}{\text{hour}}$$



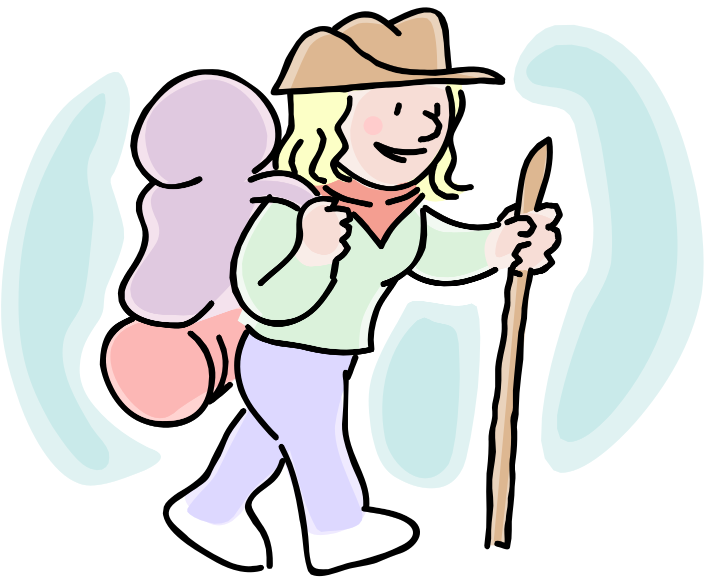
Of course, it's quite likely that we weren't walking at 2 miles/hour at every single moment. This is just an average rate over a specific time interval.

$$\text{average velocity } = \frac{4 \text{ miles}}{2 \text{ hours}} = 2\frac{\text{miles}}{\text{hour}}$$

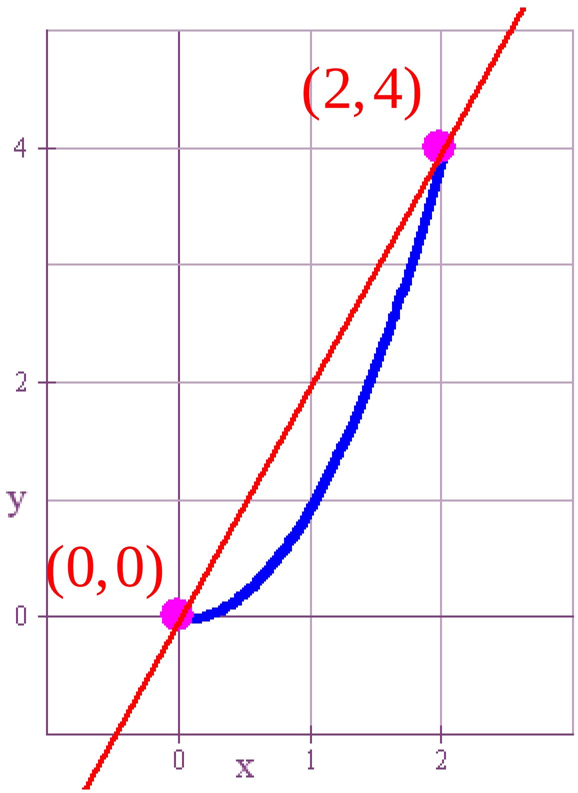


In particular, suppose our distance traveled at time *x* is given by the function $f(x)=x^2$ for *x* ranging from 0 to 2.

$$\text{average velocity} = \frac{4 \text{ miles}}{2 \text{ hours}} = 2\,\frac{\text{miles}}{\text{hour}}$$

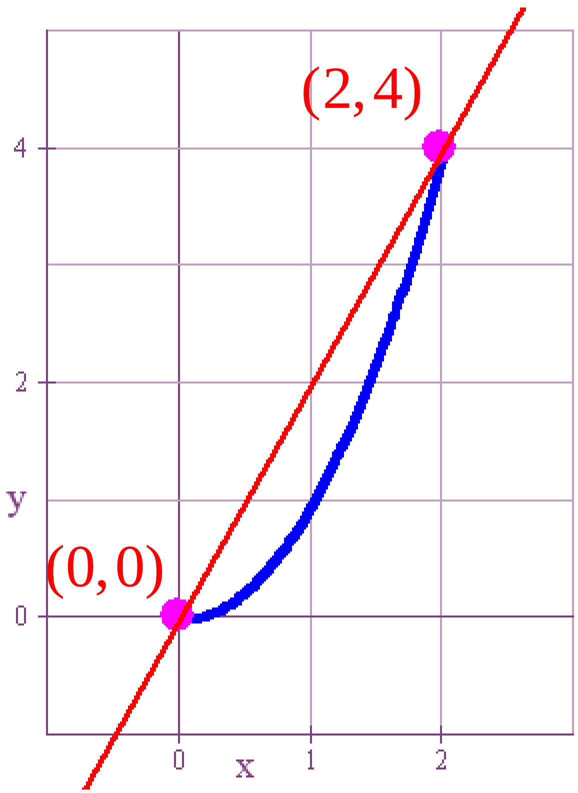


Then our average velocity is just the slope of the secant line that connects our starting point with our stopping point.



$$\text{slope} = \frac{4-0}{2-0} = \frac{4}{2} = 2$$

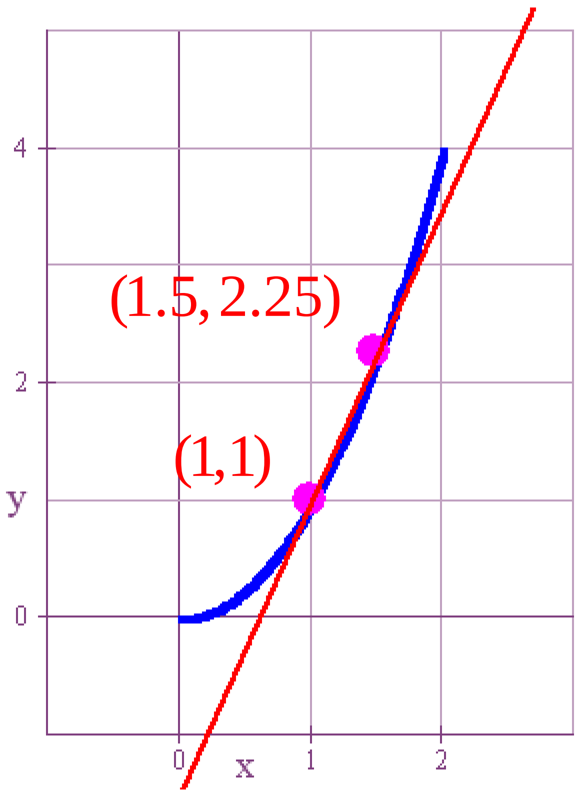
In other words, the *average rate of change* from one point on a curve to another is the same as the *slope* of the line connecting the two points.



average rate of change

$$= \text{ slope } = \frac{4-0}{2-0} = \frac{4}{2} = 2$$

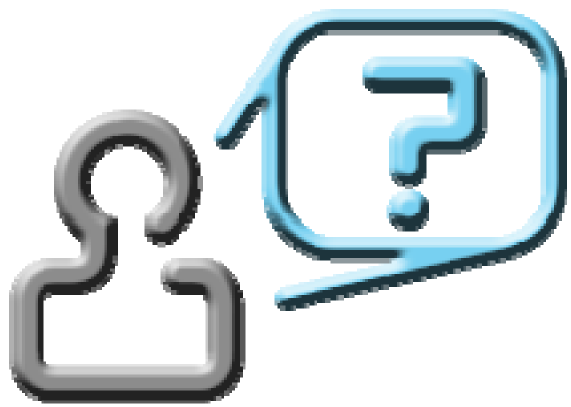
EXAMPLE: If *$f(x)=x^2$*, what is the average rate of change from *$x=1$* to *$x=1.5$*?



average rate of change

$$= \text{slope} = \frac{2.25-1}{1.5-1} = \frac{1.25}{0.5} = 2.5$$

Now suppose that the Dow Jones Industrial Average is at 13,100 on day 1 and 12,865 on day 5. What is the average rate of change?



Now suppose that the Dow Jones Industrial Average is at 13,100 on day 1 and 12,865 on day 5. What is the average rate of change?

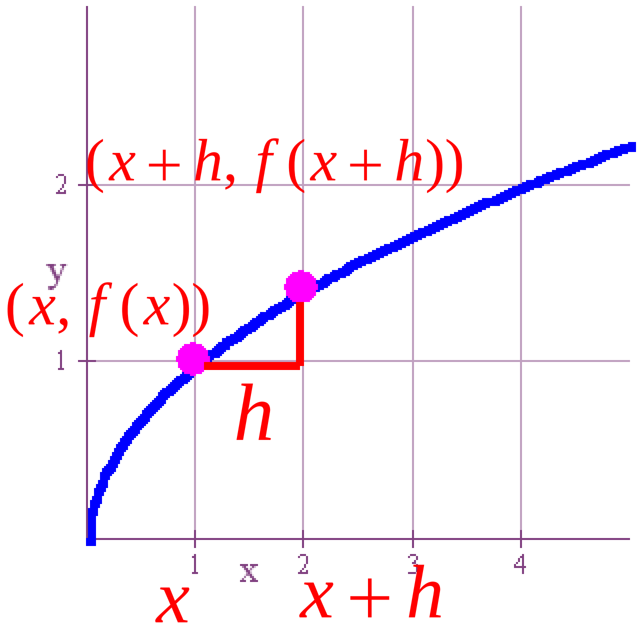
$(1, 13{,}100)$

$(5, 12{,}865)$

$$\text{average rate of change}$$

$$= \text{slope} = \frac{12865 - 13100}{5 - 1} = \frac{-235}{4} = -58.75\,\frac{\text{points}}{\text{day}}$$

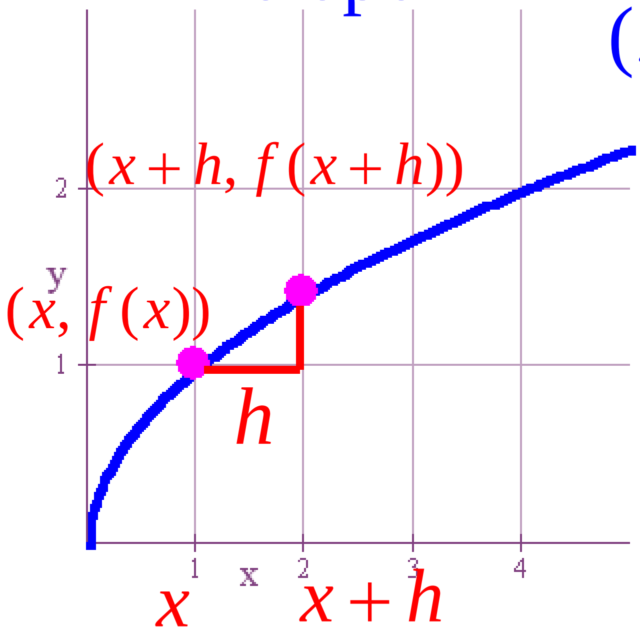
Finally, suppose we have a point on function with coordinates *(x,f(x))* and suppose also that we add an increment *h* to *x* to get a second point *(x+h,f(x+h))*.



Then the formula for the average rate of change is as follows.

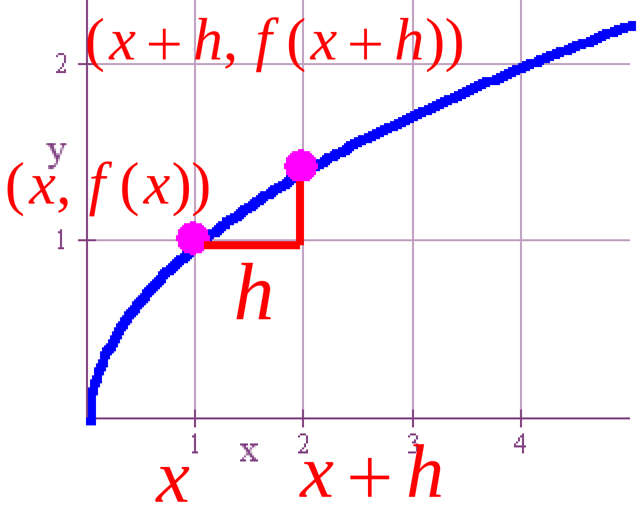
$$\text{average rate of change}$$

$$= \text{slope} = \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(x+h)-f(x)}{h}$$



This expression is also known as the *difference quotient*, and it is another way to express the average rate of change.

$$\text{average rate of change} = \text{slope} = \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(x+h)-f(x)}{h}$$



EXAMPLE: If *$f(x)=x^2$* and if we start at *$a=1$* followed by an increment of *$h=2$*, then what is the average rate of change?

$$\text{average rate of change}$$

$$= \text{slope} = \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(x+h)-f(x)}{h}$$

EXAMPLE: If *f(x)=x²* and if we start at *a=1* followed by an increment of *h=2*, then what is the average rate of change?

$$\text{average rate of change} = \text{slope} = \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(x+h)-f(x)}{h}$$

$$= \frac{f(1+2)-f(1)}{2} = \frac{f(3)-f(1)}{2} = \frac{3^2-1^2}{2} = \frac{9-1}{2} = \frac{8}{2} = 4$$