Applications of Maxima and Minima

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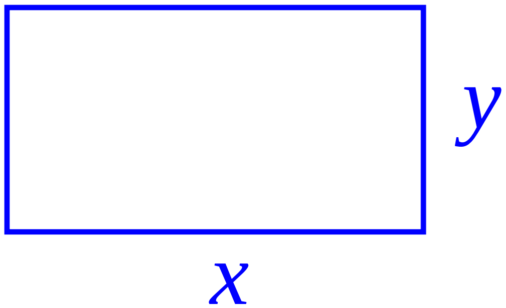
$y =$ width

$\text{Area} = xy$



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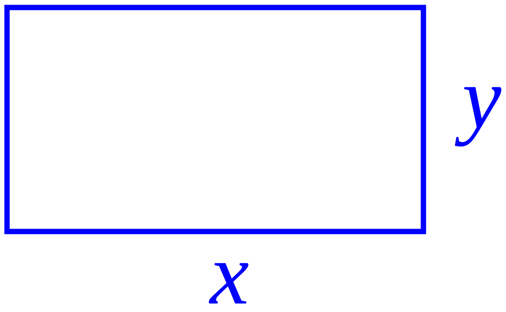

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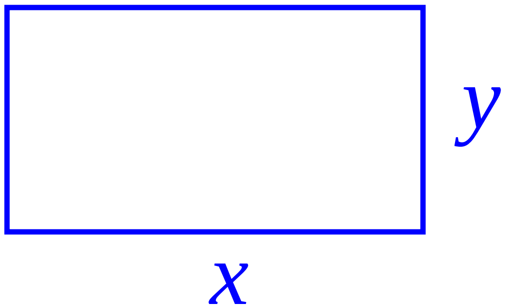

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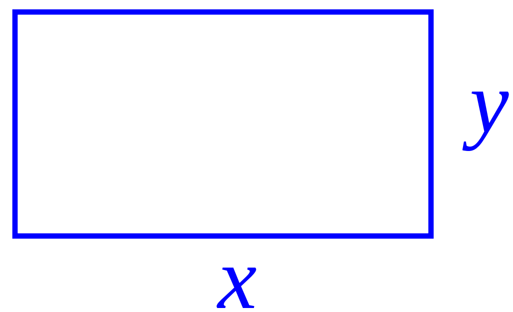

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Critical Point

$$A'(x) = 50 - 2x$$

$$50 - 2x = 0 \Rightarrow x = 25$$

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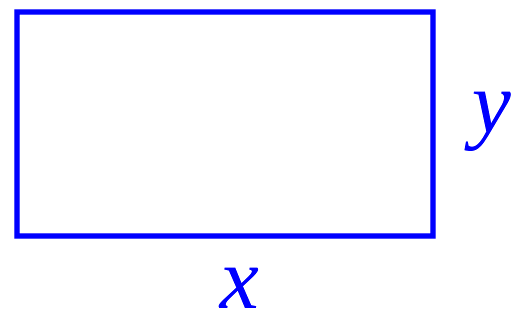

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$$A''(x) = -2$$

$$A''(25) = -2 < 0 \Rightarrow \text{ maximum}$$

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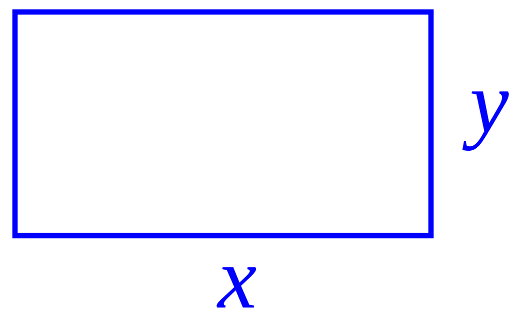

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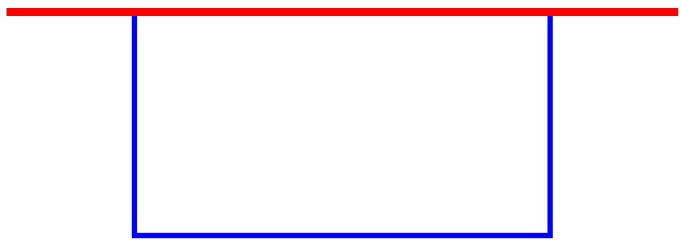
Area is maximized when both length and width are 25 feet.

Maximum area is 625 feet^2.

Now suppose you have 100 ft of fencing, but one side of the rectangular area is the side of a building. That means you only have to fence in three sides. In this situation, what is the largest rectangular area you can enclose?



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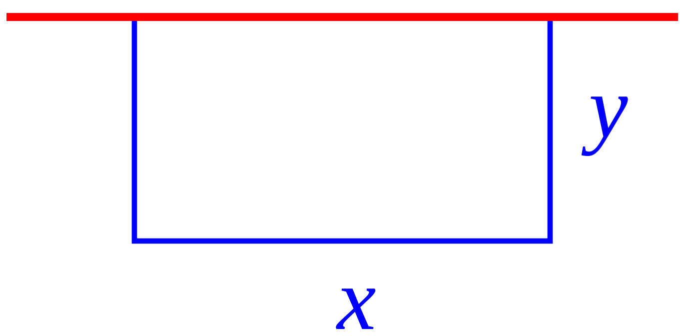

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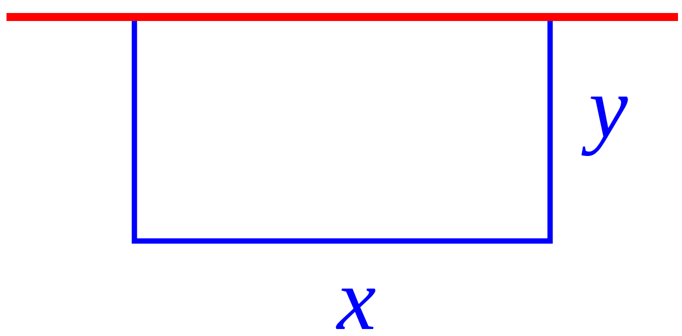
$x + 2y = 100 \Rightarrow y = 50 - x/2$



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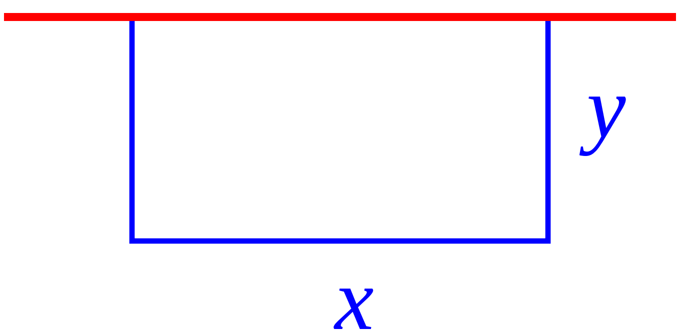

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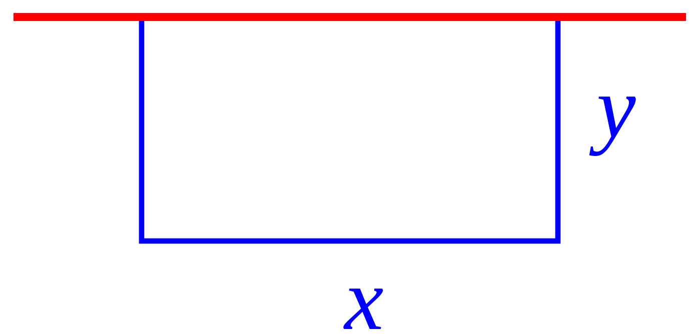

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$$A'(x) = 50 - x$$

$$50 - x = 0 \Rightarrow x = 50$$

Critical Point

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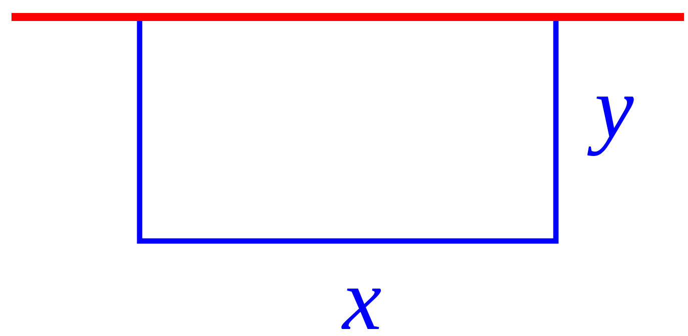

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$$A''(x) = -1$$

$$A''(50) = -1 < 0 \Rightarrow \text{ maximum}$$

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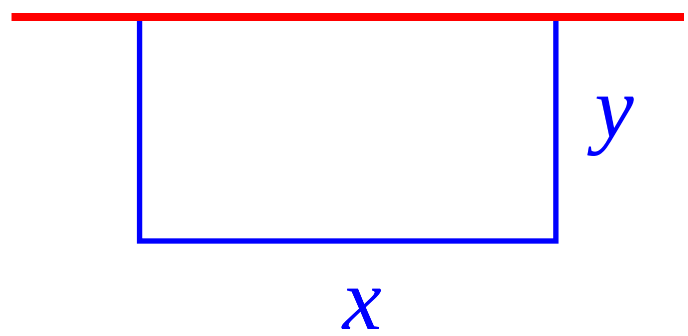

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$50 - x = 0 \Rightarrow x = 50$ ← Critical Point

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$A''(50) = -1 < 0 \Rightarrow$ maximum

Area is maximized when $x = 50$ feet and $y = 25$ feet. Maximum area is 1250 feet^2.

Hercules Films is deciding on the price of the video release of its film *Son of Frankenstein*. Its marketing people estimate that at a price of p dollars, it can sell a total of $q = 200{,}000 - 10{,}000p$ copies. What price will bring in the greatest revenue?

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$$R''(p) = -20,000 \Rightarrow R''(10) = -20,000$$

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