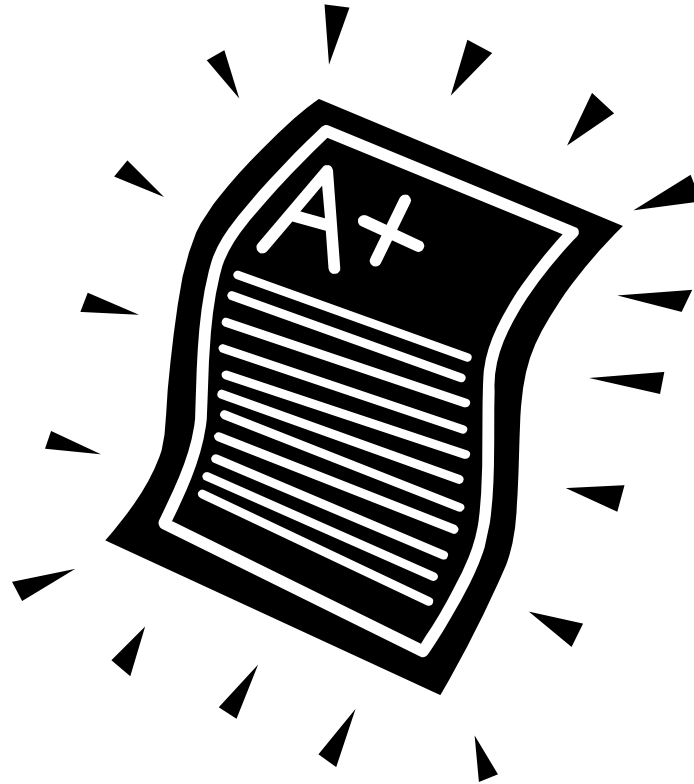


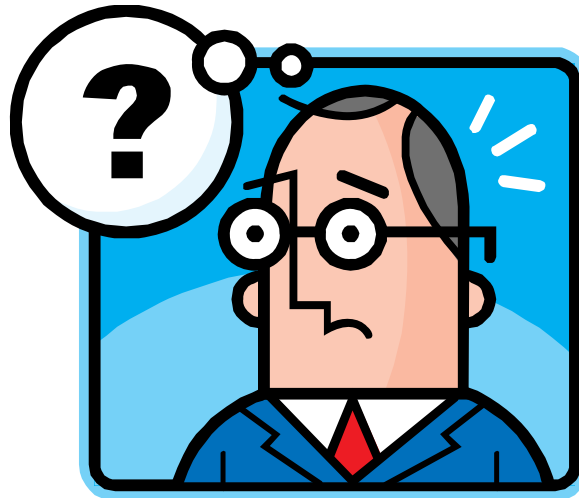
# MEASURES OF RELATIVE STANDING



Suppose in class you make a grade of 85 and in another class you get a grade of 86. Clearly, you prefer 86 over 85.



However, suppose also that the average in the first class is 82 with a standard deviation of 1 point while in the second class the average is 83 with a standard deviation of 1.5 points. Compared to your peers, in which class did you perform better?



We can compare the scores by representing each one in terms of how many standard deviations above or below the mean each one is. This is called a **standard** or **z-score**.

$$z = \frac{x - \bar{x}}{s} \quad \text{or} \quad z = \frac{x - \mu}{\sigma}$$

By converting to z-scores, we see that we did better in class 1.

$$\text{Class 1: } z = \frac{x - \bar{x}}{s} = \frac{85 - 82}{1} = \frac{3}{1} = 3$$

$$\text{Class 2: } z = \frac{x - \bar{x}}{s} = \frac{86 - 83}{1.5} = \frac{3}{1.5} = 2$$

Another way to see how we compare with our peers in a given set of numerical data is by converting our score into a **percentile**.

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The idea is to divide our data into 100 equal parts and see how many of those parts we did better than. Note, however, that you can never do better than 100% of the people since you don't perform better than yourself.

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Also, be aware that there exist slightly different procedures for dividing our data into equal parts.



If we are trying to convert a particular score to a percentile, we'll do the following:

1. Arrange the scores in order from lowest to highest.
2. Count the number of scores below the one we are converting.
3. Express this number as a percent of the whole.
4. Round to the nearest percent.
5. That's our percentile!

Here's an example:

60, 75, 78, 82, 86, 93, 95, 98

$$\frac{3}{8} = 0.375 \approx 38\%$$

82 is the 38<sup>th</sup> percentile!

We can also go in the other direction. In other words, start with a percentile and find the data value. Here's how:

1. Take the corresponding percent of the total number of scores
2. If the result is not a whole number, round to the next highest whole number. This gives you the position of the percentile.
3. If the result is a whole number, then the percentile is halfway between that position and the next number.

Here's another example. Find the 38<sup>th</sup> percentile,  $P_{38}$ .

60, 75, 78, 82, 86, 93, 95, 98

↑  
4

$$38\% \text{ of } 8 = (.38)(8) = 3.04 \approx 4$$

$$P_{38} = 82$$

Now let's find the 25<sup>th</sup> percentile,  $P_{25}$ . The 25<sup>th</sup> percentile is also known as the first quartile,  $Q_1$ .

60, 75, 78, 82, 86, 93, 95, 98

↑  
2.5

$$25\% \text{ of } 8 = (.25)(8) = 2 \rightarrow 2.5$$

$$P_{25} = Q_1 = \frac{75 + 78}{2} = 76.5$$

The 50<sup>th</sup> percentile,  $P_{50}$ , is the second quartile,  $Q_2$ , and it is the same as the median.

60, 75, 78, 82, 86, 93, 95, 98

↑  
4.5

50% of 8 =  $(.5)(8) = 4 \rightarrow 4.5$

$$P_{50} = Q_2 = \tilde{x} = \frac{82 + 86}{2} = 84$$

The 75<sup>th</sup> percentile,  $P_{75}$ , is the same as the third quartile,  $Q_3$ .

60, 75, 78, 82, 86, 93, 95, 98

↑  
6.5

75% of 8 =  $(.75)(8) = 6 \rightarrow 6.5$

$$P_{75} = Q_3 = \frac{93 + 95}{2} = 94$$

The high and the low scores plus the three quartiles give us a nice summary of the distribution of our data that we call a **5-number summary**.

60, 75, 78, 82, 86, 93, 95, 98

minimum,  $Q_1$ , median,  $Q_3$ , maximum

60, 76.5, 84, 94, 98



We can also find this summary quite easily using our calculator.

60, 75, 78, 82, 86, 93, 95, 98

L1	L2	L3	1
60	-----	-----	
75			
78			
82			
86			
93			
95			
L1(1)=60			

```

EDIT [2nd] [MODE] TESTS
1: 1-Var Stats
2: 2-Var Stats
3: Med-Med
4: LinReg(ax+b)
5: QuadReg
6: CubicReg
7: QuartReg
    
```

```

1-Var Stats
    
```

```

1-Var Stats
x̄=83.375
Σx=667
Σx²=56707
sₓ=12.51213696
σₓ=11.70403242
↓n=8
    
```

```

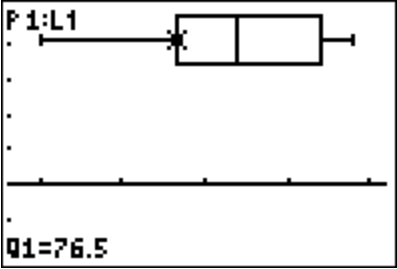
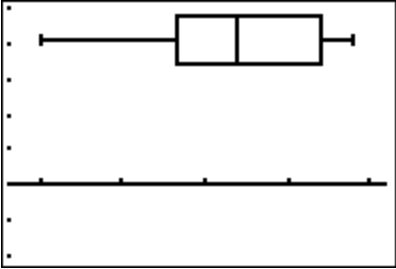
1-Var Stats
↑n=8
minX=60
Q1=76.5
Med=84
Q3=94
maxX=98
    
```

Even better, our calculator can also create a visual display of this result called a **box plot** or **box & whisker display**.

60, 75, 78, 82, 86, 93, 95, 98

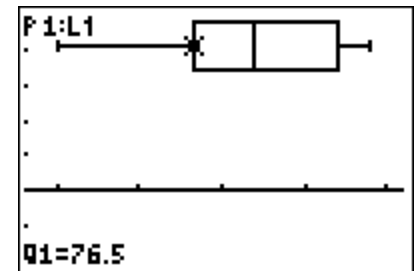
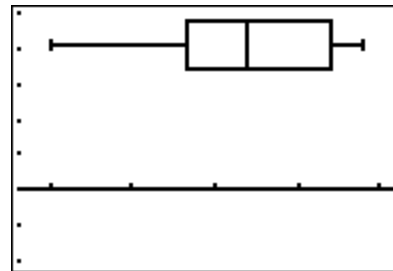
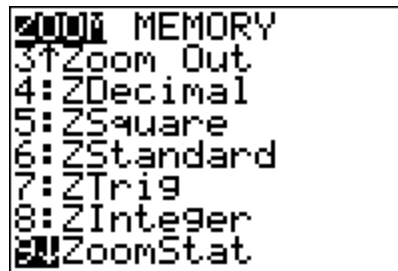
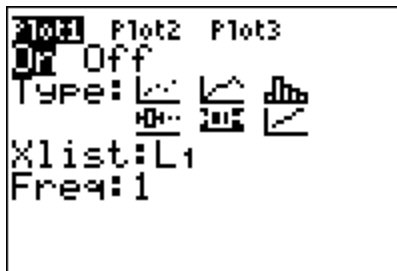
```
Plot1 Plot2 Plot3
Off Off
Type: [Line] [Bar] [Dot Plot]
Freq: [1] [2] [5] [10]
Xlist: L1
Freq: 1
```

```
MEMORY
3: Zoom Out
4: ZDecimal
5: ZSquare
6: ZStandard
7: ZTrig
8: ZInteger
9: ZoomStat
```



Here are a few more definitions.

60, 75, 78, 82, 86, 93, 95, 98



Interquartile range ==  $IQR = Q_3 - Q_1$

Semi-interquartile range =  $\frac{Q_3 - Q_1}{2}$

Midquartile =  $\frac{Q_3 + Q_1}{2}$

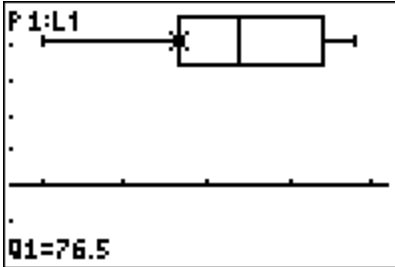
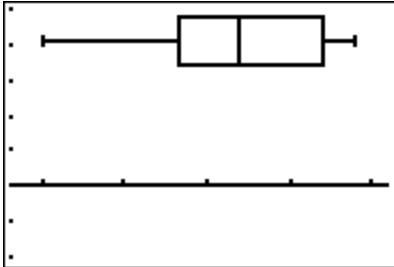
10 – 90 percentile range =  $P_{90} - P_{10}$

One definition of an outlier that many people like to use is any value that is either more than  $1.5 \times \text{IQR}$  above  $Q_3$  or more than  $1.5 \times \text{IQR}$  below  $Q_1$ .

60, 75, 78, 82, 86, 93, 95, 98

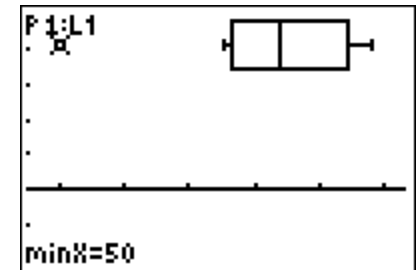
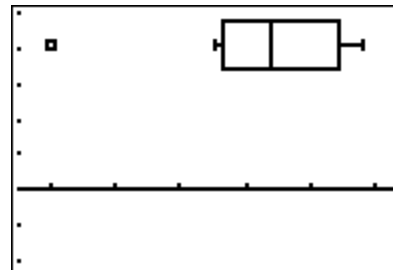
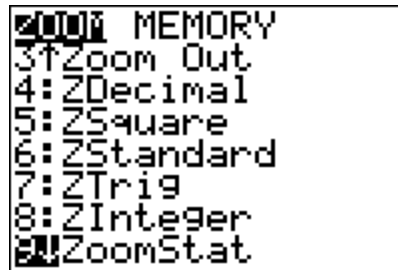
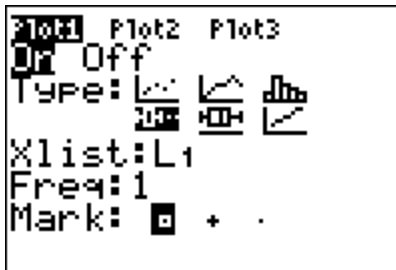
```
Plot2 Plot3
Off
Type: [line] [line] [line]
Xlist:L1
Freq:1
```

```
MEMORY
3:Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7:ZTrig
8:ZInteger
ZoomStat
```



A modified box plot lets us see if we have any outliers.

50, 75, 78, 82, 86, 93, 95, 98



$$IQR \cdot 1.5 = (Q_3 - Q_1) \cdot 1.5 = (94 - 76.5) \cdot 1.5 = 26.25$$

$$Q_1 - 26.25 = 76.5 - 26.25 = 50.25$$