PROBABILITY

We generally distinguish between three types of probability.

1. Classical or mathematical probability

2. Experimental of relative frequency probability

3. Subjective probability

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<u>Experiment:</u> We roll a fair, six sided die.

$$\text{sample space} = S = \{1, 2, 3, 4, 5, 6\}$$

$$\text{event} = E = \{2, 3\}$$

We define the probability of our event to be the ratio of favorable outcomes to total number of outcomes.

Experiment: We roll a fair, six sided die.

$$\text{sample space} = S = \{1, 2, 3, 4, 5, 6\}$$

$$\text{event} = E = \{2, 3\}$$

$$P(E) = \frac{2}{6} = \frac{1}{3} \approx 0.333$$

We cannot always determine probabilities using a purely mathematical approach. Sometimes we have to do an experiment several times to determine the frequency of a given event.

Experimental or relative frequency probability

$$P(E) = \frac{\text{number of times } E \text{ occurred}}{\text{number of times the experiment was repeated}}$$

Suppose that in a typical monsoon season that we observe that it rains 15 out of 60 days. Then we can use this historical frequency to estimate the probability of rain on any given day during the next monsoon season.

$$\text{Let } R = \text{ rain}$$

$$P(R) = \frac{15}{60} = 0.25 = 25\%$$

THE LAW OF LARGE NUMBERS: As a procedure is repeated again and again, the relative frequency probability (experimental probability) of an event tends to approach the actual probability.

In subjective probability, we cannot determine the probability mathematically and we cannot do repeated experiments to determine the expected frequency of a result. We just have to make our best, educated guess about the likelihood of a certain event.

Subjective probability must often be used when making foreign policy decisions.

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Example 1: If Country A is provoked by Country B and decides to respond with a show of force, what is the probability that Country B will back down?

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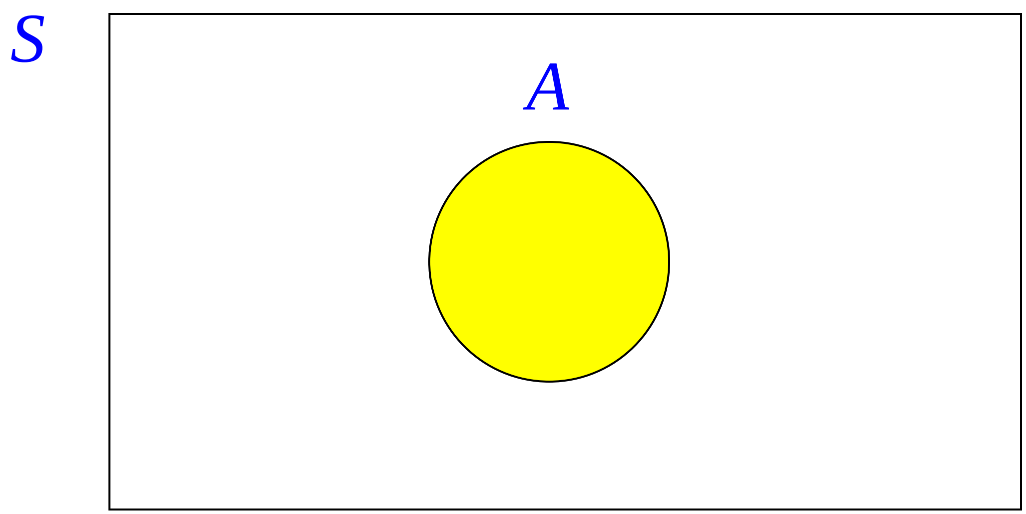
Example 2: Prior to the war in Iraq, what did Saddam Hussein think was going to happen?

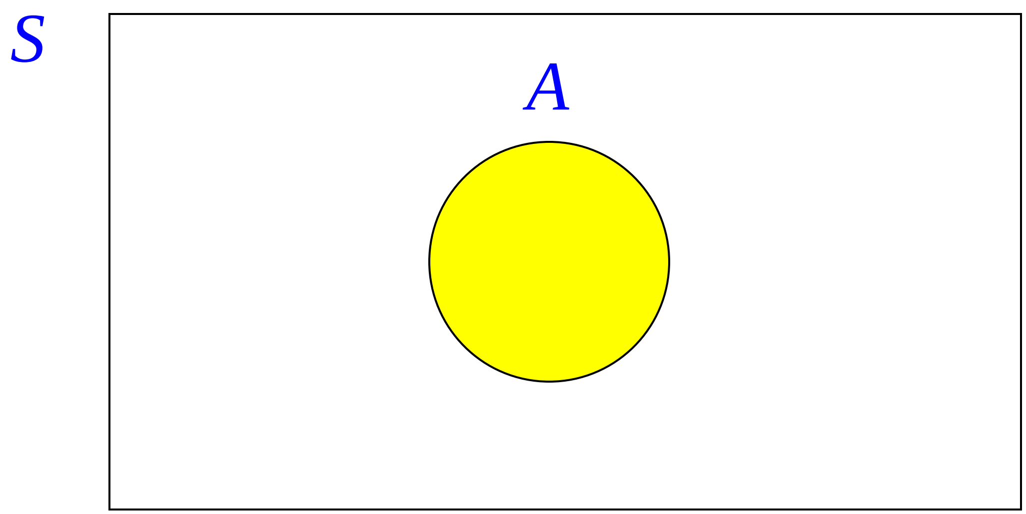
We can represent a lot of concepts of mathematical probability using *Venn diagrams*. Think of the diagram below as a dart board, and the dart can land anywhere on the board. We can compute the probability that it lands in *A* using areas.



$$P(A) = \frac{\text{area of } A}{\text{area of } S}$$

Notice that a probability always has to be between 0 and 1.

$$0 \leq P(A) \leq 1$$

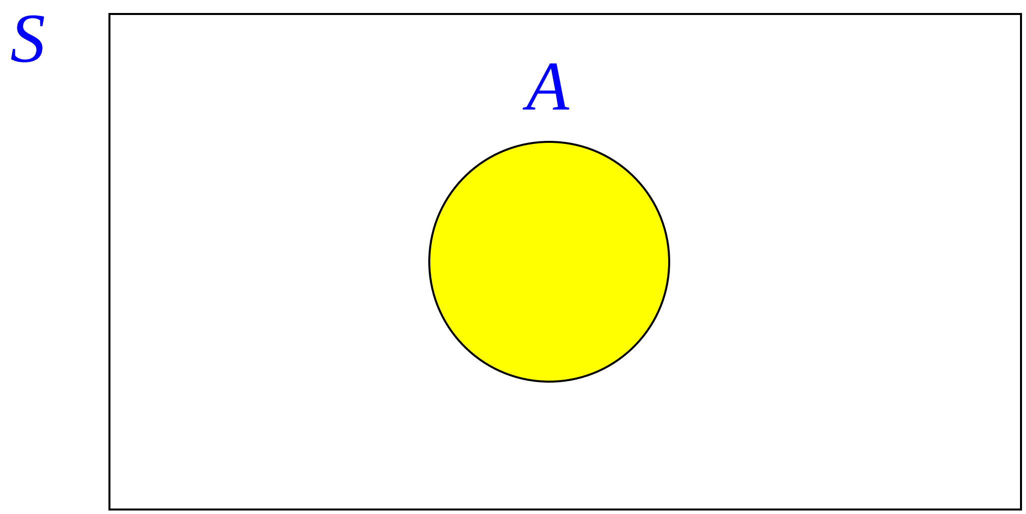


$$P(A) = \frac{\text{area of } A}{\text{area of } S}$$

We call an event with probability 1 certain and an event with probability zero impossible.

$$\text{certain event: } P(A) = 1$$

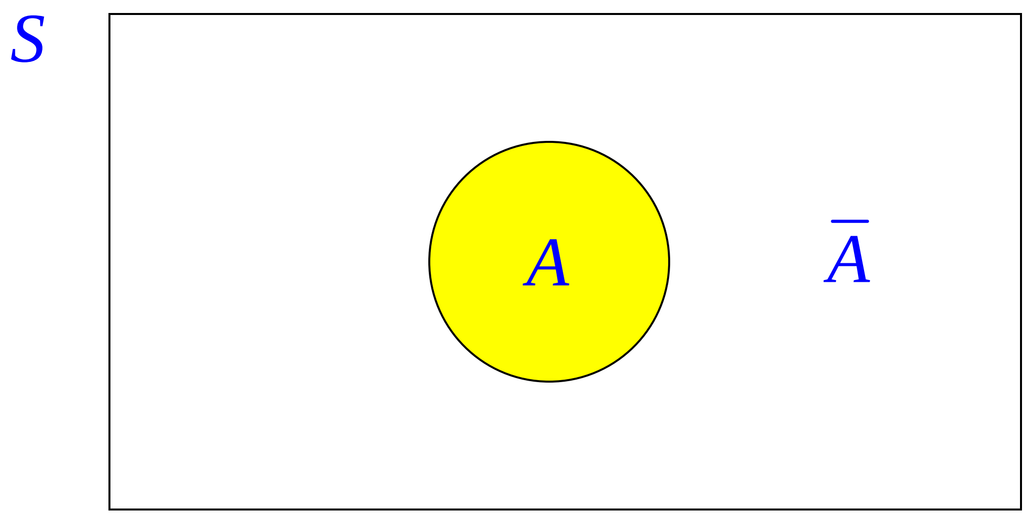
$$\text{impossible event: } P(A) = 0$$



$$P(A) = \frac{\text{area of } A}{\text{area of } S}$$

Everything in *S*, but not in *A*, we call the *complement of A.*

$$\text{A-complement} = \bar{A}$$

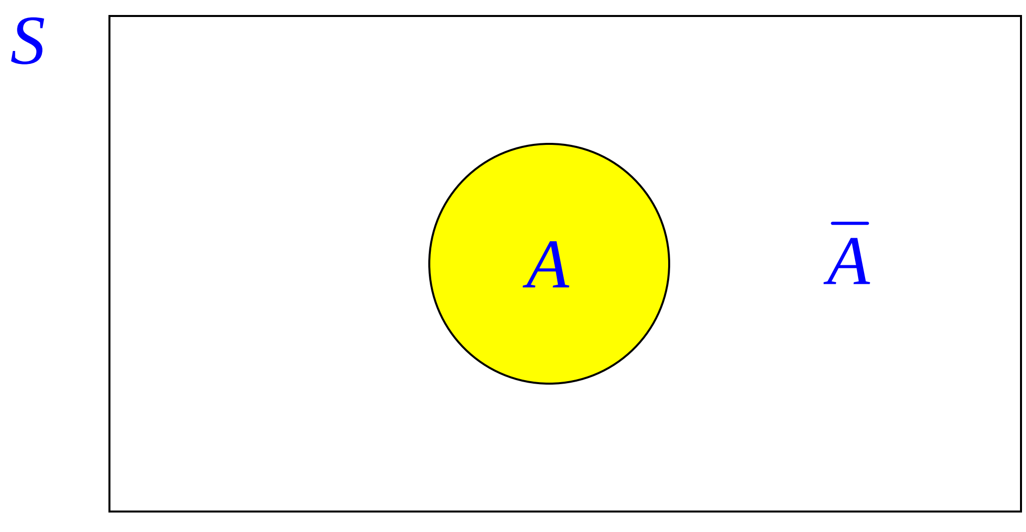


$$P(A) = \frac{\text{area of } A}{\text{area of } S}$$

Clearly,

$$1 = P(S) = \frac{\text{area of } S}{\text{area of } S} = \frac{\text{area of } A + \text{area of } \bar{A}}{\text{area of } S}$$

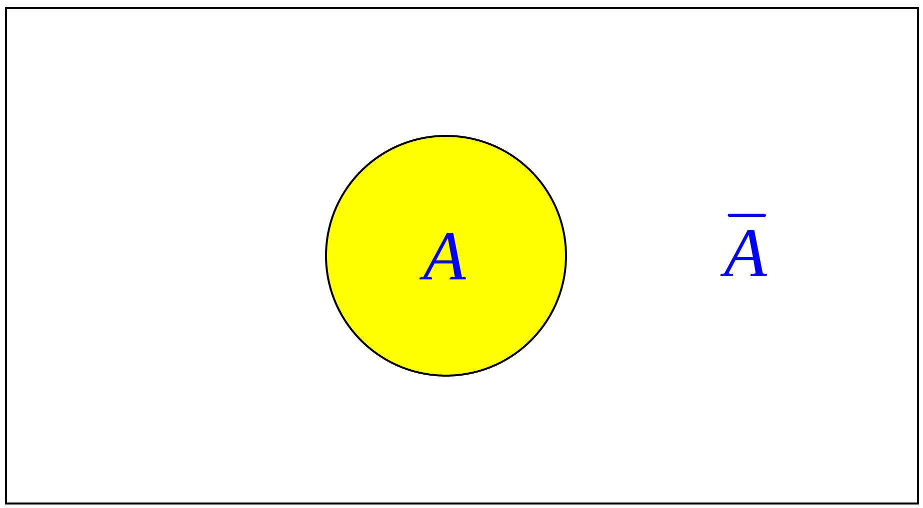
$$= \frac{\text{area of } A}{\text{area of } S} + \frac{\text{area of } \bar{A}}{\text{area of } S} = P(A) + P(\bar{A})$$



Thus,

$$P(\overline{A}) = 1 - P(A)$$

S



Example: If the probability of rain is 25%, then the probability that it won't rain is 75%.

$$P(\bar{R}) = 1 - P(R) = 1 - 0.25 = 0.75 = 75\%$$

Some odd defintions:

$$\text{odds in favor of } A = \frac{\text{number of outcomes favorable to } A}{\text{number of outcomes unfavorable to } A} = \frac{P(A)}{P(\bar{A})}$$

$$\text{odds against } A = \frac{\text{number of outcomes unfavorable to } A}{\text{number of outcomes favorable to } A} = \frac{P(\bar{A})}{P(A)}$$

$$\text{payoff odds} = \frac{\text{net profit}}{\text{amount bet}}$$

Suppose the probability of rain, *R*, is 25%.

$$\text{odds in favor of } R = \frac{P(R)}{P(\overline{R})} = \frac{0.25}{0.75} = 1:3$$

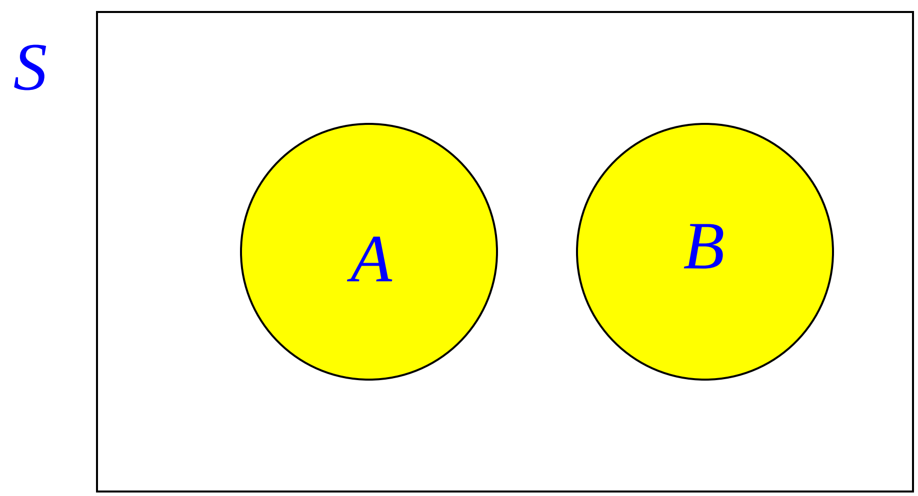
$$\text{odds against } R = \frac{P(\overline{R})}{P(R)} = \frac{0.75}{0.25} = 3:1$$

Suppose you bet $5 on number 13 in roulette, and the payoff odds are 35:1. How much might you win?

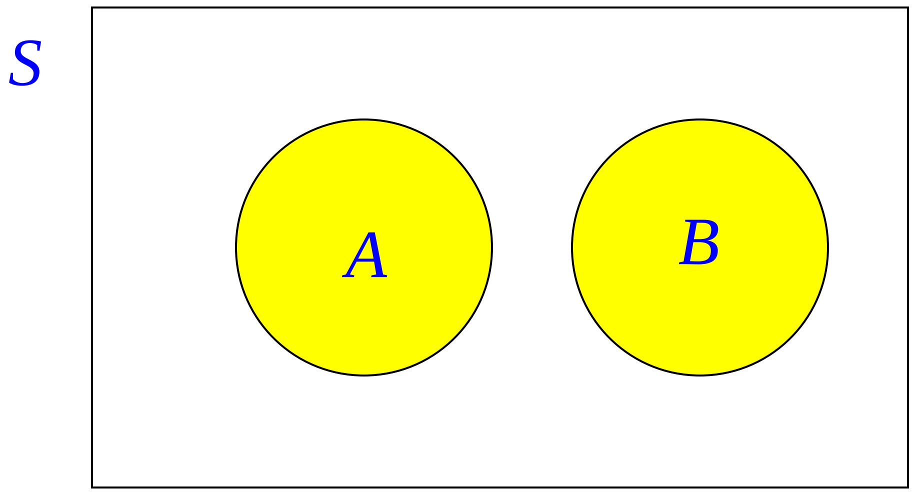
$$\text{payoff odds} = 35{:}1$$

$$\text{potential winnings} = (35)(5) = \$175$$

Now suppose that on our dart board we have two regions, *A* and *B*.



In this case, the intersection of *A* and *B* is empty.

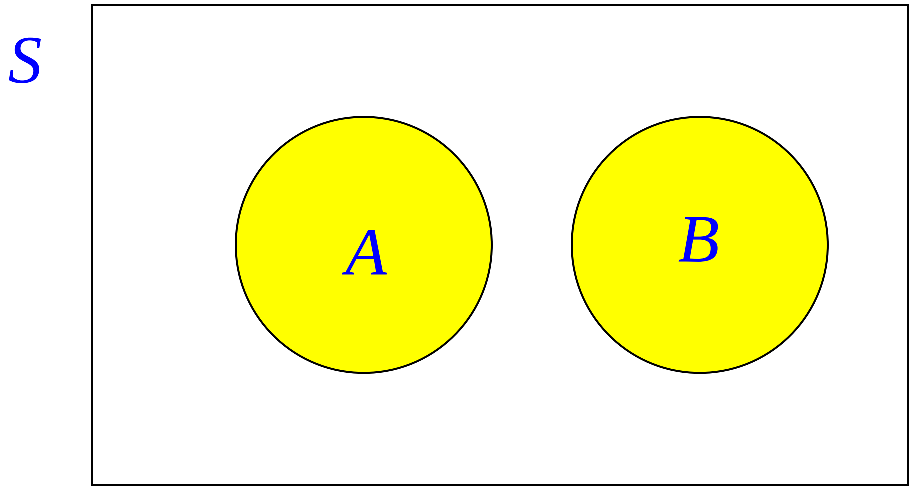


$$A \cap B = \varnothing$$

We can easily compute the probability that a dart lands in *A* or *B*, though, using areas.

$$P(A \text{ or } B) = P(A \cup B) = \frac{\text{area of } A + \text{area of } B}{\text{area of } S}$$

$$= \frac{\text{area of } A}{\text{area of } S} + \frac{\text{area of } B}{\text{area of } S} = P(A) + P(B)$$



$$A \cap B = \varnothing$$

In this situation we say that *A* and *B* are *disjoint* or that the two events are *mutually exclusive*.

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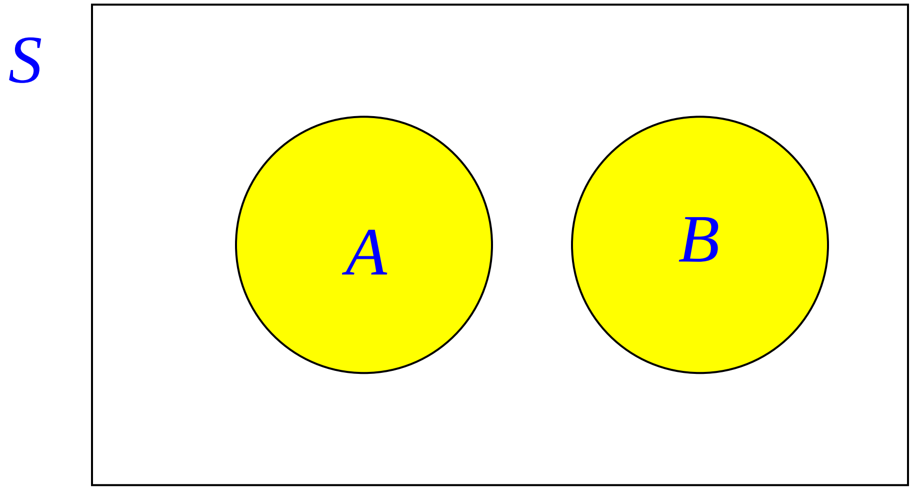
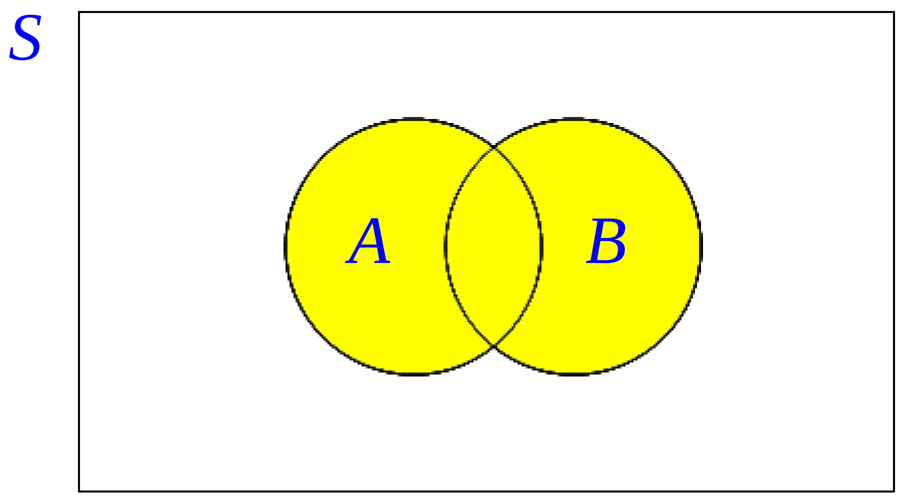


$$A \cap B = \varnothing$$

If our events are not *mutually exclusive*, then the formula is a little different.

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - P(A \,\&\, B)$$



$$A \cap B \neq \varnothing$$

Experiment 1: We roll a fair, six sided die.

$$\text{sample space} = S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{1, 2, 3\}$$

$$B = \{4, 5\}$$

$$A \cap B = \varnothing$$

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

Experiment 2: We roll a fair, six sided die.

$$\text{sample space} = S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{1, 2, 3\}$$

$$B = \{2, 3, 4\}$$

$$A \cap B \neq \varnothing$$

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \,\&\, B)$$

$$= P(A) + P(B) - P(A \cap B) = \frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$$