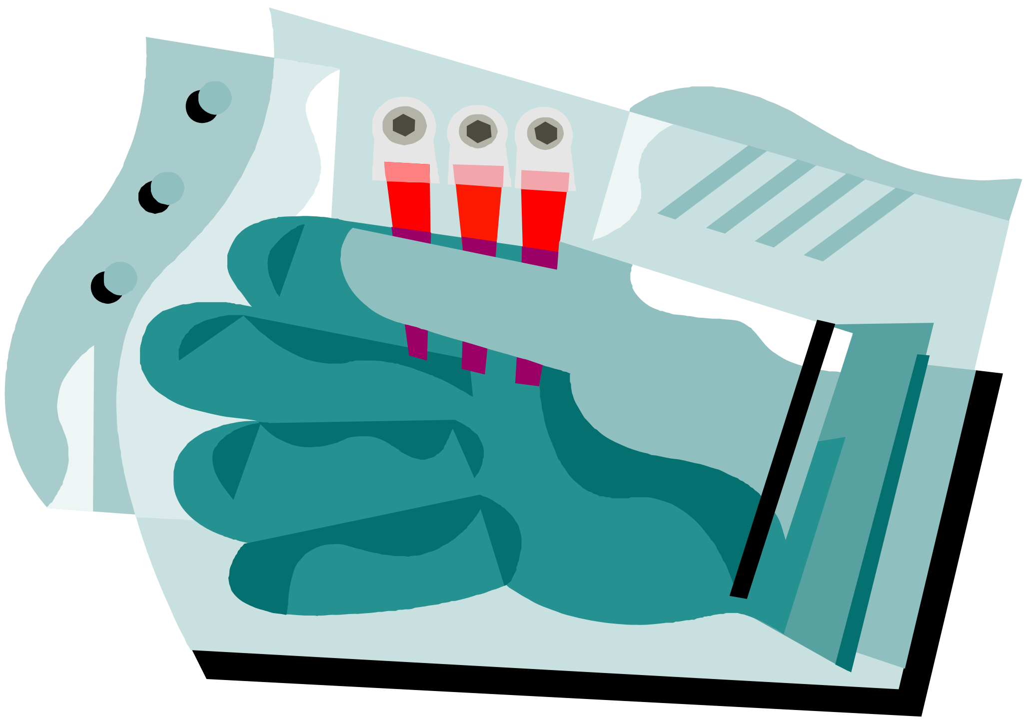
SAMPLING DISTRIBUTIONS



The sampling distribution of a statistic is the distribution of all values of the statistic when all possible samples of the same size *n* are taken from the same population.

The sampling distribution of the mean is the distribution of sample means when all possible samples of the same size *n* are taken from the same population.

$$\bar{x} = \text{ sample mean}$$

$$\mu = \text{ population mean}$$

The sampling distribution of the variance is the distribution of sample variances when all possible samples of the same size *n* are taken from the same population.

$$s^2 = \text{sample variance}$$

$$\sigma^2 = \text{population variance}$$

The sampling distribution of the proportion is the distribution of sample proportions when all possible samples of the same size *n* are taken from the same population.

$$\hat{p} = \text{ sample proportion}$$

$$p = \text{ population proportion}$$

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In this case, we say that the sample mean, sample variance, and sample proportion *target* the population mean, variance, and proportion.

We also say that each of these is an *unbiased estimator* of the corresponding population parameter.

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The *median* and the *range* are also *not unbiased estimators*. They do not target the corresponding population parameters as the sample size increases.

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On the other hand, the sampling distribution of the sample variances tends toward a distribution that is skewed to the right as the sample size increases.

The Central Limit Theorem: Let X be a random variable with mean μ and standard deviation σ, and consider the sampling distribution of the sample means for all simple random samples of size *n*. We will denote the mean of this distribution by $\mu_{\overline{x}}$ and the standard deviation by $\sigma_{\overline{x}}$. Then,

1. $\mu_{\overline{x}} = \mu$
2. $\sigma_{\overline{x}} = \dfrac{\sigma}{\sqrt{n}}$ (the standard error of the mean)
3. The sampling distribution of the sample means approaches a normal distribution as the sample size increases.

Additionally, we have following two results:

1. If the parent population is normally distributed, then the sampling distribution of the sample means will always be normally distributed, regardless of the sample size *n*.

2. If the parent population is *<u>not</u>* normally distributed, then we do not expect the sampling distribution of the sample means to reasonably approximate a normal distribution unless $n > 30$.

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What is the probability that an *individual* man has a weight greater than 185 lb?

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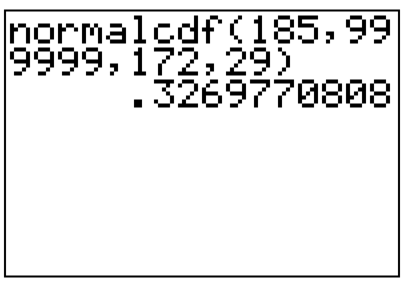
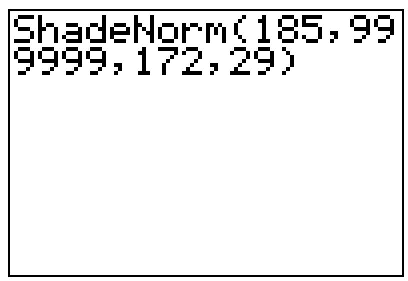
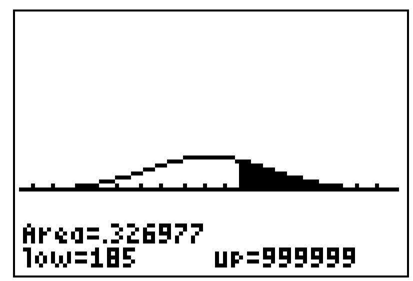
$$z = \frac{x - \mu}{\sigma} = \frac{185 - 172}{29} \approx 0.448$$







Suppose that weights of men are normally distributed with mean 172 lb and standard deviation 29 lb.

What is the probability that *20 randomly selected men* have a mean weight greater than 185 lb?

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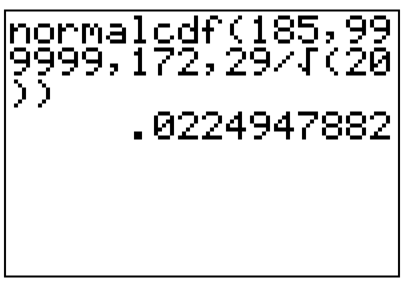
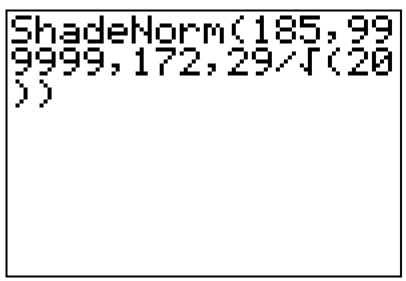
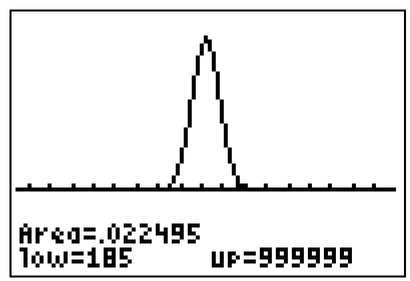
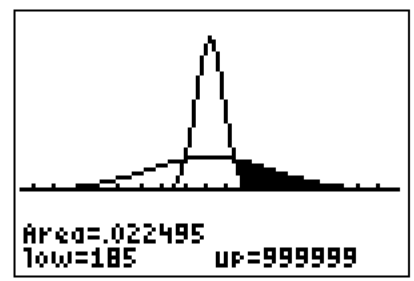
$$z = \frac{\overline{x} - \mu_{\overline{x}}}{\sigma_{\overline{x}}} = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}} = \frac{185 - 172}{29 / \sqrt{20}} \approx 2.005$$









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normalcdf(12.04,
999999,12,.11/6)
          .0145614254
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ShadeNorm(12.04,
999999,12,.11/6)
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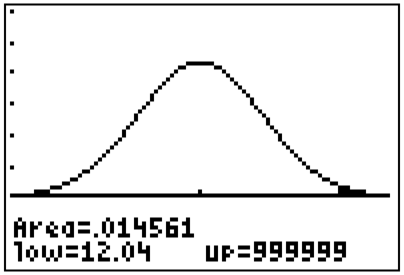

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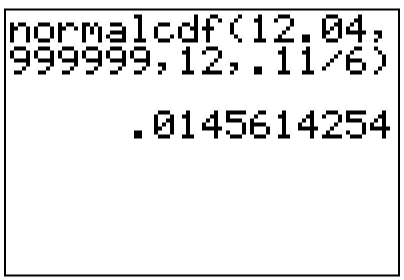

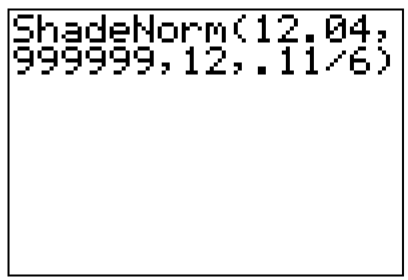

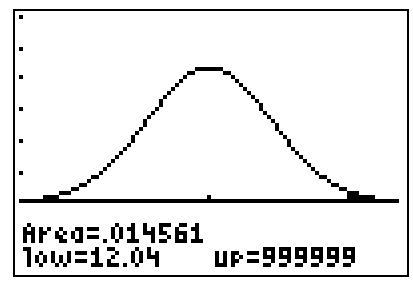

Does this difference have practical significance to you?