PRACTICE – WHAT IS A GROUP? – ANSWERS

1. Explain why the set of real numbers under subtraction does not form a group.

   The set of real numbers under subtraction does not obey the associative law. In other words, $(3-2)-1=1-1=0$, but $3-(2-1)=3-1=2$. Thus, $(3-2)-1\neq 3-(2-1)$.

2. Explain why the set of real numbers under multiplication does not form a group.

   Because the identity in the set of real numbers under multiplication is 1, it follows that 0 is an element in this set that has no multiplicative inverse. Hence, we don't have a group.

3. Explain why the set of irrational numbers under multiplication does not form a group.

   There are two reasons this set under multiplication is not a group. First, it is not closed since $\sqrt{2}\cdot\sqrt{2}=2$, and 2 is not irrational, and second, the set of irrational numbers under multiplication does not contain an identity element.

4. <u>Prove:</u> The identity element *e* in a group *G* is unique.

   <u>Proof:</u> Suppose $e_1, e_2 \in G$ are both identity elements in a group *G*. Then $e_1 = e_1 e_2 = e_2$.
   □

5. <u>Prove:</u> If *G* is a group and $a \in G$, then *a* has only one unique inverse.

   <u>Proof:</u> Suppose that *G* is a group and $a \in G$ has two inverses which we'll denote by a^{-1} and b^{-1}. Then

   $$a^{-1}a = e = b^{-1}a \Rightarrow a^{-1}a = b^{-1}a \Rightarrow (a^{-1}a)a^{-1} = (b^{-1}a)a^{-1} \Rightarrow a^{-1}(aa^{-1}) = b^{-1}(aa^{-1})$$
   $$\Rightarrow a^{-1}e = b^{-1}e \Rightarrow a^{-1} = b^{-1}.$$

   Therefore, in a group the inverse of an element is unique. □