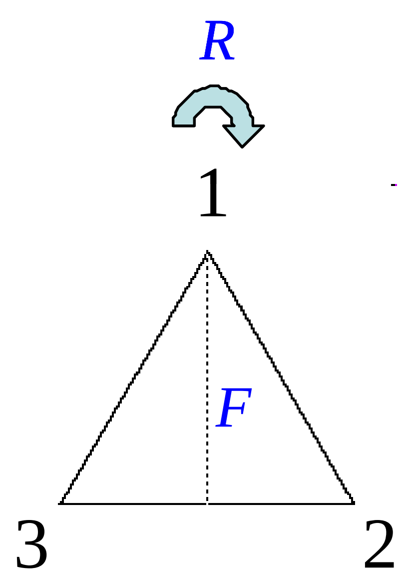
PRACTICE – SUBGROUPS – ANSWERS

1. The group of symmetries of the equalateral triangle, S_3, has six subgroups, one of which is the trivial subgroup $\{e\}$ and another is the whole group itself. Use the multiplication table below to find the remaining four subgroups.

	$(1)(2)(3)$	$(1\ 2)$	$(1\ 3)$	$(2\ 3)$	$(1\ 2\ 3)$	$(1\ 3\ 2)$
$(1)(2)(3)$	$(1)(2)(3)$	$(1\ 2)$	$(1\ 3)$	$(2\ 3)$	$(1\ 2\ 3)$	$(1\ 3\ 2)$
$(1\ 2)$	$(1\ 2)$	$(1)(2)(3)$	$(1\ 2\ 3)$	$(1\ 3\ 2)$	$(1\ 3)$	$(2\ 3)$
$(1\ 3)$	$(1\ 3)$	$(1\ 3\ 2)$	$(1)(2)(3)$	$(1\ 2\ 3)$	$(2\ 3)$	$(1\ 2)$
$(2\ 3)$	$(2\ 3)$	$(1\ 2\ 3)$	$(1\ 3\ 2)$	$(1)(2)(3)$	$(1\ 2)$	$(1\ 3)$
$(1\ 2\ 3)$	$(1\ 2\ 3)$	$(2\ 3)$	$(1\ 2)$	$(1\ 3)$	$(1\ 3\ 2)$	$(1)(2)(3)$
$(1\ 3\ 2)$	$(1\ 3\ 2)$	$(1\ 3)$	$(2\ 3)$	$(1\ 2)$	$(1)(2)(3)$	$(1\ 2\ 3)$



The remaining subgroups are $\left\{\begin{matrix}(1)(2)(3)\\(1\ 2)\end{matrix}\right\}$, $\left\{\begin{matrix}(1)(2)(3)\\(1\ 3)\end{matrix}\right\}$, $\left\{\begin{matrix}(1)(2)(3)\\(2\ 3)\end{matrix}\right\}$, and $\left\{\begin{matrix}(1)(2)(3)\\(1\ 2\ 3)\\(1\ 3\ 2)\end{matrix}\right\}$.

2. For each subgroup of order 2 or 3 in S_3, find the left coset created by multiplying the subgroup on the left by the permutation $(1\quad 2\quad 3)$.

$$(1\quad 2\quad 3)\begin{Bmatrix}(1)(2)(3)\\(1\quad 2)\end{Bmatrix}=\begin{Bmatrix}(1\quad 2\quad 3)\\(2\quad 3)\end{Bmatrix}$$

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$$(1\quad 2\quad 3)\begin{Bmatrix}(1)(2)(3)\\(2\quad 3)\end{Bmatrix}=\begin{Bmatrix}(1\quad 2\quad 3)\\(1\quad 3)\end{Bmatrix}$$

$$(1\quad 2\quad 3)\begin{Bmatrix}(1)(2)(3)\\(1\quad 2\quad 3)\\(1\quad 3\quad 2)\end{Bmatrix}=\begin{Bmatrix}(1\quad 2\quad 3)\\(1\quad 3\quad 2)\\e\end{Bmatrix}$$

3. <u>Prove:</u> S_3 has no subgroup of order 5.

<u>Proof:</u> The order of S_3 is $|S_3| = 3! = 3\cdot 2\cdot 1 = 6$, and 5 is not a divisor of 6. Hence, by Lagrange's Theorem, S_3 cannot have a subgroup of order 5. $\{e\}\square$

4. <u>Prove:</u> For any group *G*, $\{e\}$ is a normal subgroup.

<u>Proof:</u> Let *G* be a group and let $a \in G$. Then $aea^{-1} = aa^{-1} = e \in \{e\}$. Therefore, $\{e\}$ is a normal subgroup of *G*, $\{e\} \triangleleft G$. $\square$