

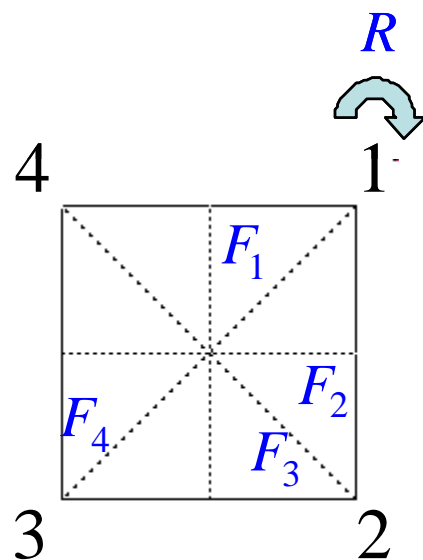
PRACTICE – SOME SPECIAL CASES OF GROUPS – ANSWERS

1. Complete the multiplication table below for \mathbb{Z}_5 .

	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

2. With reference to the square diagram below, complete the following multiplication table for D_4 .

	(1)(2)(3)(4)	(1 2 3 4)	(1 3)(2 4)	(1 4 3 2)	(1 4)(2 3)	(1 2)(3 4)	(1 3)	(2 4)
(1)(2)(3)(4)	(1)(2)(3)(4)	(1 2 3 4)	(1 3)(2 4)	(1 4 3 2)	(1 4)(2 3)	(1 2)(3 4)	(1 3)	(2 4)
(1 2 3 4)	(1 2 3 4)	(1 3)(2 4)	(1 4 3 2)	(1)(2)(3)(4)	(1 3)	(2 4)	(1 2)(3 4)	(1 4)(2 3)
(1 3)(2 4)	(1 3)(2 4)	(1 4 3 2)	(1)(2)(3)(4)	(1 2 3 4)	(1 2)(3 4)	(1 4)(2 3)	(2 4)	(1 3)
(1 4 3 2)	(1 4 3 2)	(1)(2)(3)(4)	(1 2 3 4)	(1 3)(2 4)	(2 4)	(1 3)	(1 4)(2 3)	(1 2)(3 4)
(1 4)(2 3)	(1 4)(2 3)	(2 4)	(1 2)(3 4)	(1 3)	(1)(2)(3)(4)	(1 3)(2 4)	(1 4 3 2)	(1 2 3 4)
(1 2)(3 4)	(1 2)(3 4)	(1 3)	(1 4)(2 3)	(2 4)	(1 3)(2 4)	(1)(2)(3)(4)	(1 2 3 4)	(1 4 3 2)
(1 3)	(1 3)	(1 4)(2 3)	(2 4)	(1 2)(3 4)	(1 2 3 4)	(1 4 3 2)	(1)(2)(3)(4)	(1 3)(2 4)
(2 4)	(2 4)	(1 2)(3 4)	(1 3)	(1 4)(2 3)	(1 4 3 2)	(1 2 3 4)	(1 3)(2 4)	(1)(2)(3)(4)



3. With respect to the diagram below, complete the following alternate multiplication table for D_4 .

	e	R	R^2	R^3	F	FR^2	FR^3	FR
e	e	R	R^2	R^3	F	FR^2	FR^3	FR
R	R	R^2	R^3	e	FR^3	FR	FR^2	F
R^2	R^2	R^3	e	R	FR^2	F	FR	FR^3
R^3	R^3	e	R	R^2	FR	FR^3	F	FR^2
F	F	FR	FR^2	FR^3	e	R^2	R^3	R
FR^2	FR^2	FR^3	F	FR	R^2	e	R	R^3
FR^3	FR^3	F	FR	FR^2	R	R^3	e	R^2
FR	FR	FR^2	FR^3	F	R^3	R	R^2	e

Use that,

$$\begin{aligned}
 e &= (1)(2)(3)(4) \\
 R &= (1\ 2\ 3\ 4) \\
 R^2 &= (1\ 3)(2\ 4) \\
 R^3 &= (1\ 4\ 3\ 2) \\
 F_1 = F &= (1\ 4)(2\ 3) \\
 F_2 = FR^2 &= (1\ 2)(3\ 4) \\
 F_3 = FR^3 &= (1\ 3) \\
 F_4 = FR &= (2\ 4) \\
 FR = R^3F &\ \&\ RF = FR^3
 \end{aligned}$$

