PRACTICE 1 – PERMUTATIONS – ANSWERS

In problems 1 through 5, assume we are talking about permutations of the numbers, 1, 2, 3, and 4.

1. Write $(1 \quad 4 \quad 2 \quad 3)$ as a product of transpositions. Is $(1 \quad 4 \quad 2 \quad 3)$ an even permutation or an odd permutation?

   $(1 \quad 4 \quad 2 \quad 3) = (1 \quad 4)(1 \quad 2)(1 \quad 3)$ is an odd permutation.

2. Write $(1 \quad 4 \quad 2 \quad 3)$ in the form $\begin{pmatrix} 1 & 2 & 3 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ ? & ? & ? & ? \end{pmatrix}$.

   $(1 \quad 4 \quad 2 \quad 3) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 3 & 1 & 2 \end{pmatrix}$

3. Express the permutation $(1 \quad 4 \quad 2 \quad 3)$ as a 4×4 permutation matrix times the column matrix $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$.

   $$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 1 \\ 2 \end{pmatrix}$$

4. Multiply $(1 \quad 4 \quad 2 \quad 3)(2 \quad 3 \quad 1)$. Remember to multiply left to right. Also, classify the result as either an even permutation or an odd permutation.

   $(1 \quad 4 \quad 2 \quad 3)(2 \quad 3 \quad 1) = (1 \quad 4 \quad 3 \quad 2) = (1 \quad 4)(1 \quad 3)(1 \quad 2)$ is an odd permutation.

5. Express $(1\ \ 4\ \ 2\ \ 3)(2\ \ 3\ \ 1)$ as a product of permutation matrices, and verify that it produces the correct permutation of the numbers $\begin{pmatrix}1\\2\\3\\4\end{pmatrix}$.

$$(1\ \ 4\ \ 2\ \ 3)(2\ \ 3\ \ 1)=\begin{pmatrix}0&0&0&1\\0&0&1&0\\1&0&0&0\\0&1&0&0\end{pmatrix}\begin{pmatrix}0&1&0&0\\0&0&1&0\\1&0&0&0\\0&0&0&1\end{pmatrix}=\begin{pmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&1&0\end{pmatrix}$$

$$\begin{pmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&1&0\end{pmatrix}\begin{pmatrix}1\\2\\3\\4\end{pmatrix}=\begin{pmatrix}4\\1\\2\\3\end{pmatrix}$$

6. Complete the multiplication table below for the products of the permutations of the numbers 1, 2, and 3. Remember to multiply left to right.

	$(1)(2)(3)$	$(1\ \ 2)$	$(1\ \ 3)$	$(2\ \ 3)$	$(1\ \ 2\ \ 3)$	$(1\ \ 3\ \ 2)$
$(1)(2)(3)$	$(1)(2)(3)$	$(1\ \ 2)$	$(1\ \ 3)$	$(2\ \ 3)$	$(1\ \ 2\ \ 3)$	$(1\ \ 3\ \ 2)$
$(1\ \ 2)$	$(1\ \ 2)$	$(1)(2)(3)$	$(1\ \ 2\ \ 3)$	$(1\ \ 3\ \ 2)$	$(1\ \ 3)$	$(2\ \ 3)$
$(1\ \ 3)$	$(1\ \ 3)$	$(1\ \ 3\ \ 2)$	$(1)(2)(3)$	$(1\ \ 2\ \ 3)$	$(2\ \ 3)$	$(1\ \ 2)$
$(2\ \ 3)$	$(2\ \ 3)$	$(1\ \ 2\ \ 3)$	$(1\ \ 3\ \ 2)$	$(1)(2)(3)$	$(1\ \ 2)$	$(1\ \ 3)$
$(1\ \ 2\ \ 3)$	$(1\ \ 2\ \ 3)$	$(2\ \ 3)$	$(1\ \ 2)$	$(1\ \ 3)$	$(1\ \ 3\ \ 2)$	$(1)(2)(3)$
$(1\ \ 3\ \ 2)$	$(1\ \ 3\ \ 2)$	$(1\ \ 3)$	$(2\ \ 3)$	$(1\ \ 2)$	$(1)(2)(3)$	$(1\ \ 2\ \ 3)$

By the way, notice that each row and each column contains all the possible permutations with no repetitions. As we'll see later, this is no accident!