

## PRACTICE – CONJUGATES AND COMMUTATORS

1. If  $a = \begin{pmatrix} 1 & 7 \\ & \end{pmatrix}$  and  $b = \begin{pmatrix} 1 & 2 & 3 \\ & & \end{pmatrix}$ , find the conjugate  $aba^{-1}$ .
2. If  $a = \begin{pmatrix} 1 & 7 \\ & \end{pmatrix}$  and  $b = \begin{pmatrix} 1 & 2 & 3 \\ & & \end{pmatrix}$ , find the commutator  $aba^{-1}b^{-1}$ .
3. Prove: If  $G$  is a group (not necessarily finite),  $H$  is a subgroup of  $G$ , and  $a \in G$ , then  $aHa^{-1}$  is also a subgroup of  $G$ .
4. Prove: If  $G$  is a group (not necessarily finite),  $a \in G$ , and  $x \in G$  such that the order of  $\langle x \rangle$  in  $G$  is  $|\langle x \rangle| = n$ , then the order of the conjugate of  $x$ ,  $axa^{-1}$ , is also  $n$ .
5. Prove: If  $G$  is a group (not necessarily finite), and  $a, b \in G$ , then  $aba^{-1} = e$  if and only if  $b = e$ .